Even-Odd Mode Analysis of Unequal Dual-Band Wilkinson Power Divider

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1. INTRODUCTION

The power divider/combiner is very important component for power amplifier design in low power MMIC design and high power system using hybrid scheme. In addition to that, the power divider/combiner is applied for impedance matching, antenna polarization, and phase control in phased array antenna. In this paper, we will consider the power divider/combiner working at two arbitrary frequencies with different power ratios at two output ports. In order to determine the design parameters, the even-odd mode analysis is carried out rigorously using impedance matching condition and lumped components such as R, L, and C. The validation of analysis and design parameters is verified using two commercially available softwares based on circuit theory and FIT algorithm, respectively.

2. EVEN-ODD MODE ANALYSIS

A. Even-mode analyses at port 2 and 3

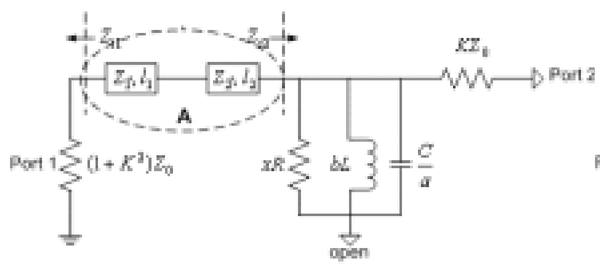


Fig.1. Equivalent circuit for the even mode analysis at port 2

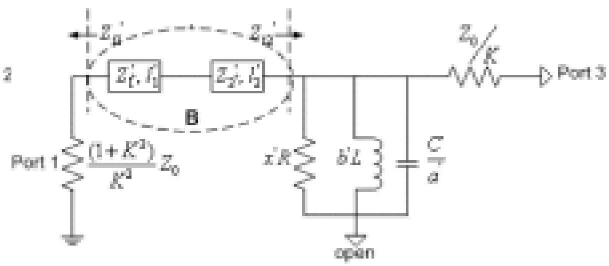


Fig.2. Equivalent circuit for the even mode analysis at port 3

The equivalent circuit of even-mode power divider is shown in Fig. 1. Since the signals traveling into port 2 and port 3 have the equal amplitude and equal phase, respectively, the effects of R, L and C can be ignored due to the voltage balance between two output ports. Therefore, both impedances seen looking into the left and right from the transmission line (denoted by “A”) consisting of two different characteristic impedances Z_1 and Z_2 can be written as following :

$$Z_{i1} = (1+K^2)Z_0,\ Z_{i2} = KZ_0 \tag{1}$$

where K^2 means power ratio. In this case, Z_{i1} and Z_{i2} have been determined by using the even-mode analysis of single-band unequal Wilkinson power divider. Using eq. (1) and the procedure of impedance matching condition at two arbitrary frequencies f_1 and f_2 described in [1], the lengths (l_1 and l_2) of two different transmission lines and characteristic impedances (Z_1 and Z_2) are determined as follows.

$$l_1 = l_2 = \frac{n\pi}{\beta_1 + \beta_2}$$

$$Z_2 = Z_0\sqrt{\frac{K}{2\alpha}(K^2 - K + 1) + \sqrt{\left[\frac{K}{2\alpha}(K^2 - K + 1)\right]^2 + K^3(1 + K^2)}}$$

$$Z_1 = \frac{K(1 + K^2)Z_0^2}{Z_2}$$

where $\alpha = \left(\tan\left(\beta_1 l_1\right)\right)^2$.

In a similar way as shown in section 2-A, the equivalent circuit of even-mode at port 3 can be described as shown in Fig. 2 without contribution of the lumped elements, R, L, and C. The impedances seen looking into left and right from the transmission line (denoted by “B”) are

$$Z_{i1}^{'} = \frac{(1 + K^2)}{K^2} Z_0 \text{ and } Z_{i2}^{'} = Z_0 / K,$$

respectively. The above impedances and the impedance matching condition lead the following design parameters at even-mode analysis.

$$l_1^{'} = l_2^{'} = \frac{n\pi}{\beta_1 + \beta_2}$$

$$Z_2^{'} = Z_0\sqrt{\frac{(K^2 - K + 1)}{2\alpha K^3} + \sqrt{\left[\frac{(K^2 - K + 1)}{2\alpha K^3}\right]^2 + \frac{1 + K^2}{K^5}}}$$

$$Z_1^{'} = \frac{Z_0^2}{Z_2^{'}}\frac{1 + K^2}{K^3}$$

Hence, we can obtain the relationship between the impedances at port 2 and port 3.

$$Z_2^{'} = Z_2 / K^2, \quad Z_1^{'} = Z_1 / K^2 \tag{2}$$

B. Odd-mode analyses at port 2 and 3

In this case, the lumped elements, R, L, and C contribute the electrical behaviors to matching the impedances seen looking into the left from port 2. The impedance, Z_{io2} must be equal to KZ_0 to obtain a low-reflection coefficient.

$$Z_{io2} = \left(\frac{1}{Z_{io1}} + \frac{1}{xR} + j\left(\omega_1\frac{C}{a} - \frac{1}{\omega_1 bL}\right)\right)^{-1} = KZ_0$$

From the mathematical manipulation of above equation, we can derive the relationship of the design parameters as functions of characteristic impedances, length of transmission line, power ratio and arbitrary frequencies.

$$R = \frac{KZ_0}{x}, \quad L = \frac{\dfrac{\omega_1}{\omega_2 b} - \dfrac{\omega_2}{\omega_1 b}}{\omega_2 A - \omega_1 B}, \quad C = \frac{\dfrac{A}{\omega_2} - \dfrac{B}{\omega_1}}{\dfrac{\omega_1}{a\omega_2} - \dfrac{\omega_2}{a\omega_1}},$$

where

$$A = \frac{Z_2 - Z_1 \tan^2 \beta_1 l_1}{Z_2 (Z_1 + Z_2) \tan \beta_1 l_1}, \quad B = \frac{Z_2 - Z_1 \tan^2 \beta_2 l_1}{Z_2 (Z_1 + Z_2) \tan \beta_2 l_1}.$$

In a similar way as shown in case of odd-mode analysis at port 2, we can obtain the optimized value of R, L, and C parameters for odd-mode analysis at port 3 as follows.

$$R = \frac{Z_0}{x' K}, \quad L = \frac{\dfrac{\omega_1}{\omega_2 b'} - \dfrac{\omega_2}{\omega_1 b'}}{\omega_2 A' - \omega_1 B'}, \quad C = \frac{\dfrac{A'}{\omega_2} - \dfrac{B'}{\omega_1}}{\dfrac{\omega_1}{a'\omega_2} - \dfrac{\omega_2}{a'\omega_1}}$$

where

$$A' = \frac{Z_2' - Z_1' \tan^2 \beta_1 l_1}{Z_2' (Z_1' + Z_2') \tan \beta_1 l_1}, \quad B' = \frac{Z_2' - Z_1' \tan^2 \beta_2 l_1}{Z_2' (Z_1' + Z_2') \tan \beta_2 l_1}.$$

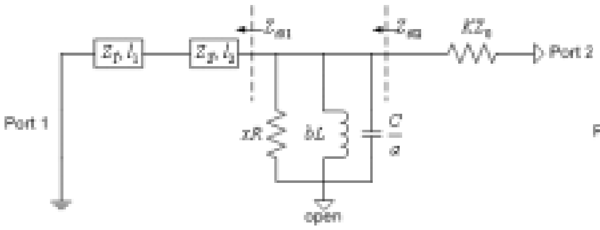


Fig.3. Equivalent circuit for the odd-mode analysis at port 2

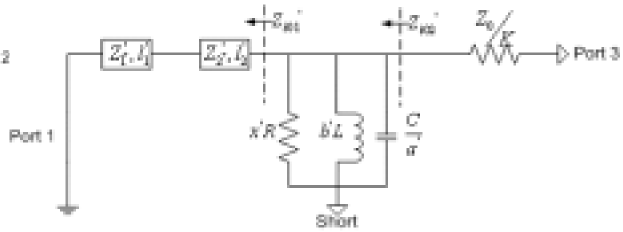


Fig.4. Equivalent circuit for the odd-mode analysis at port 2

In two cases of odd-mode analyses in Fig. 3 and 4, the relationship between the primed and unprimed parameters should satisfy the following rule that summation of two values is equal to one.

$$x' + x = 1, \quad b' + b = 1, \quad a' + a = 1 \tag{3}$$

Finally, the above relationship and eq. (1) lead to the following solution, which represents the design parameters as functions of power ratio, two arbitrary frequencies, characteristic impedance and the lengths of transmission lines.

$$R = \frac{1+K^2}{K} Z_0, \quad L = \frac{1+K^2}{K} \left(\frac{\dfrac{\omega_1}{\omega_2} - \dfrac{\omega_2}{\omega_1}}{\omega_2 A - \omega_1 B} \right), \quad C = \frac{K^2}{1+K^2} \left(\frac{\dfrac{A}{\omega_2} - \dfrac{B}{\omega_1}}{\dfrac{\omega_1}{\omega_2} - \dfrac{\omega_2}{\omega_1}} \right).$$

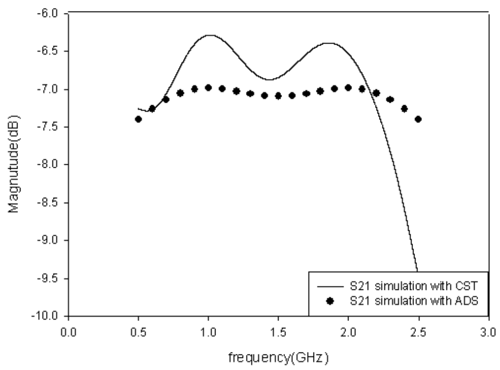
3. Verification of Design Parameters

According to the analysis described in Section II, the calculated values of design parameter are listed in Table 1 as a function of the variation of power ratio. The examples are to design 1:2, 1:3, and 1:4 unequal dual-band Wilkinson power divider operating at two arbitrary frequencies, $f_1 = 1GHz$ and

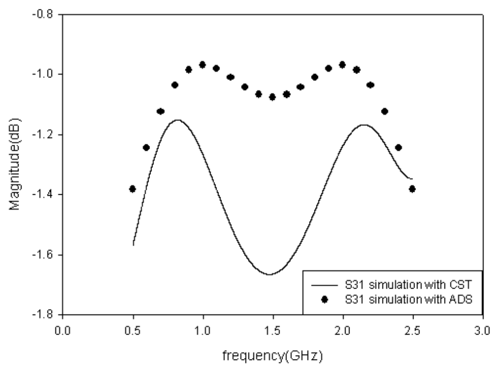
$f_2 = 2GHz$ using commercially available softwares, ADS and CST MW Studio.

Table 1. The calculated values of design parameters

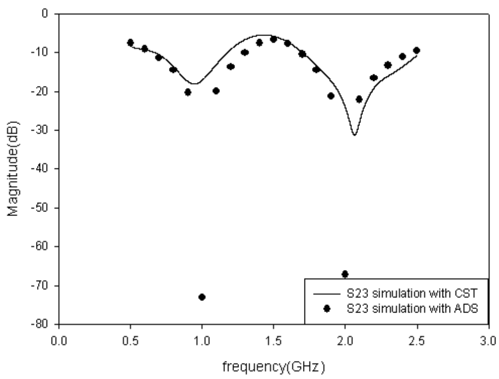
	$Z_1\,(\Omega)$	$Z_2\,(\Omega)$	$Z_1^{'}(\Omega)$	$Z_2^{'}(\Omega)$	$R\ (\Omega)$	$L\ (nH)$	$C\ (pF)$
1:2	116.59	90.9754	58.2938	45.4877	106.07	15.09	0.84
1:3	151.04	114.68	50.35	38.23	115.47	16.55	0.77
1:4	183.76	136.04	45.94	34.01	125	18.05	0.70



(a)



(b)



(c)

Fig. 5. Comparison results of 1:4 unequal Wilkinson power divider (a) S21 (b) S31 (c) S23(for isolation)

4. CONCLUSION

A rigorous analysis for Wilkinson power divider has been shown to be capable of unequal dual-band operation with no restriction of power ratio and frequency separation. In addition, the validation for design parameters have been carried out by numerical analysis and simulator-based results.

REFERENCES

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